



## COURSE DESCRIPTION CARD - SYLLABUS

Course name

Modeling of discrete and continuous systems [S2MiBM2>MUDiC2]

### Course

Field of study

Mechanical Engineering

Year/Semester

1/2

Area of study (specialization)

Virtual Engineering Design

Profile of study

general academic

Level of study

second-cycle

Course offered in

Polish

Form of study

full-time

Requirements

compulsory

### Number of hours

Lecture

15

Laboratory classes

15

Other

0

Tutorials

0

Projects/seminars

0

### Number of credit points

2,00

### Coordinators

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### Lecturers

### Prerequisites

1. Basic knowledge of mechanics, strength of materials, and mathematics corresponding to the core curriculum for the first-cycle studies. 2. The ability to solve elementary problems in mechanics and strength of materials to the extent consistent with the core curriculum of the first-cycle studies. 3. The ability to independently study and develop skills using the indicated literature, Internet sources and any other available opportunities. 4. Awareness of the need to constantly learn and improve one's skills.

### Course objective

1. Getting to know and understanding the basic concepts, laws, and equations of theory of elasticity used to create a mathematical model of linear and nonlinear mechanical discrete systems. 2. The ability to apply the acquired knowledge to modeling engineering problems, critically analyzing the results of numerical simulations and reasoning.

### Course-related learning outcomes

Knowledge:

1. A student who has completed the course knows and is able to explain the basic concepts of the theory of elasticity regarding discrete mechanical systems with constraints.

2. The student knows the principles of creating mathematical models of equilibrium and motion of basic structural elements.

Skills:

1. The student is able to analyze the state of stress and strain in the range of finite and infinitesimal strains.
2. He is able to formulate initial-boundary problems of the theory of elasticity related to typical applications of mechanical engineering.
3. He is able to critically analyze the results of numerical simulations.
4. He understands the index notation which is commonly used in solid mechanics and he can employ it making simple calculations.

Social competences:

1. The student understands the importance of knowledge in the modern world. He is also well aware that the rapid development of knowledge causes the need for lifelong learning.
2. He can organise the processes of learning and self-education.

### Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Lectures: The written exam consists of a theoretical part and a practical one consisting in modeling the issues of balance and motion of continuous mechanical systems and analyzing the states of stress and strain. The total number of points is divided into theoretical and task parts in the proportion of 30%:70%.

The condition for passing is obtaining at least 50% of the expected number of points; the grading scale is linear.

Laboratory classes: assessment of studies prepared by students based on tasks completed during classes. The required elements of the study are: Mathematica's script created during the classes, analysis and discussion of the obtained results.

The knowledge necessary to complete the task and the ability to solve the problem are assessed.

Assessment on a point scale. The condition for passing is obtaining at least 50% of the total number of points; the grading scale is linear.

Grades: very good - if the ratio of sums of achieved and total points is bigger than 90,1%; good plus - if the ratio of sums of achieved and total points is between 80,1-90%; good - if the ratio of sums of achieved and total points is between 70,1-80%; satisfactory plus - if the ratio of sums of achieved and total points is between 60,1-70%; satisfactory - if the ratio of sums of achieved and total points is between 50,1-60%; if the sum is smaller than 50% - unsatisfactory.

### Programme content

Lectures:

Material continuum.

Tensor physical quantities used in the mechanics of continuous systems.

Elements of tensor calculus. Basic principles of index notation.

Transformation and rotation of the coordinate system – transformation of coordinates of vectors and tensors.

Modelling of forces acting on a solid body.

Analysis of the stress state.

Kinematics of continuous systems.

Deformation of bodies and tensor measures of deformation.

Strain and tensor measures of strain.

Anisotropy. Constitutive relations of linear elasticity theory.

Equations of motion of continuous systems.

Initial-boundary problems of linear elasticity theory.

Plane stress state and plane strain state.

Laboratory classes:

Rotation matrix. Transformation of coordinates of vectors and tensors when the coordinate system is rotated. Orthogonal transformations.

Analysis of the three-dimensional stress state with determination of eigenvalues and eigenvectors.

Analysis of the three-dimensional strain state with determination of eigenvalues.  
Formulation and solution of selected initial-boundary problems of continuous mechanics.

## Course topics

### Lectures:

Parameters and position of points of continuous systems as continuous functions of time and spatial variables. Material continuum as a model of a deformable body.

Scalar, vector and tensor physical quantities. Selected properties of second-order tensors and actions on second-order tensors.

Absolute, analytical and index notation in the mechanics of continuous systems. Basic principles of index notation and Einstein's summation convention.

Translation of the coordinate system. Rotation of the coordinate system. Transformation of versors, coordinates of vectors and tensors when the coordinate system is rotated. Rotation matrix.

Point and continuously distributed forces. Density of continuously distributed forces. The Cauchy stress vector. The Piola-Kirchhoff stress vector. Modelling forces acting on a solid body.

Stress state at a point. The Cauchy stress tensor. The Cauchy relations between stress tensor coordinates and stress vector coordinates. Tangential and normal components of the stress vector.

Principal stresses and principal directions of the stress tensor. Determination of principal stresses and principal directions of the stress tensor. Invariants of the Cauchy stress tensor. Solving problems of the stress state analysis.

Maximum tangential stresses and the planes on which they occur. The problem of determining maximum tangential stresses as an optimization problem.

Introduction to the kinematics of continuous systems. Body configuration and natural configuration. Lagrangian (material) and Eulerian (spatial) descriptions. Function of motion and its properties. Velocity and acceleration of body particles in the material and spatial description. Displacement vector.

Deformation of a body. Deformation gradient. Displacement vector gradient. Strains. Volumetric and shear strain. Measures of local strain. Green's strain tensor. Interpretation of the components of the Green's strain tensor. Analysis of the state of strain. Infinitesimal strain tensor. Axiator and deviator. Analysis of the state of strain. Principal axes of the Green's tensor and the infinitesimal strain tensor.

Anisotropy, material symmetry and isotropy of mechanical properties of continuous systems. Generalized Hooke's law for an anisotropic body. Hooke's law for an isotropic body. Lamé's constants and engineering constants.

Material derivative. Law of continuity of mass. Law of motion for a deformable body. Dynamic problems of linear elasticity theory. Displacement equations. Initial conditions. Boundary conditions of the first, second and third kind. Mixed boundary conditions. Symmetry conditions. Plane strain state and plane stress state.

### Laboratory classes:

Tensors. Rotation matrix. Coordinate transformation of tensors and vectors when the coordinate system rotates – orthogonal transformations.

Analysis of the three-dimensional stress state. Cauchy stress formula. Determination of eigenvalues and eigenvectors of the stress state.

Kinematics of a continuous system. Material and spatial velocity field.

Deformation gradient and displacement vector gradient tensors. Analysis of three-dimensional strain state. Determination of eigenvalues and eigenvectors for the strain state.

Formulating and solving of selected initial-boundary problems of the mechanics of continuous systems.

### Teaching methods

Lectures: lecture aided by multimedia presentations, solving tasks at the blackboard, and discussion.  
Laboratory classes: solving selected problems regarding coordinate transformation, stress and strain analysis using Mathematica tools for symbolic and numerical calculations. Creating mathematical models of equilibrium and motion problems of selected continuous mechanical systems (strings, rods, beams), solving model equations using Mathematica functions, presenting the results in graphical form and analyzing the obtained results.

An online course is available on the e-Kursy platform, supporting and supplementing the teaching process carried out during lectures and laboratories.

### Bibliography

Basic:

1. Sprężystość, Mechanika techniczna t. IV, red. M. Sokołowski, PWN.
2. T. Chmielewski, S. Imielowski, Wybrane zagadnienia teorii sprężystości i plastyczności, OWPW, 2018.

Additional:

1. W. Nowacki, Teoria sprężystości, PWN.
2. G. E. Mase, Theory and problems of continuum mechanics, McGraw Hil, 1970.
3. F. M. Capaldi, Constitutive Modeling of Structural and Biological Materials, Cambridge University Press, 2012.

### Breakdown of average student's workload

	Hours	ECTS
Total workload	50	2,00
Classes requiring direct contact with the teacher	32	1,50
Student's own work (literature studies, preparation for laboratory classes/ tutorials, preparation for tests/exam, project preparation)	18	0,50